Coherent two pion photoproduction on ^{12}C

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Abstract

We develop the formalism for coherent two pion photoproduction in nuclei and perform actual calculations of cross sections for $\pi^-\pi^+$ and $\pi^0\pi^0$ photoproduction on ^{12}C. We find that due to the isospin symmetry the cross section for $\pi^0\pi^0$ production is very small and has a maximum when the pions propagate together. However, the kinematical region where the energies and polar angles of the two π^0 mesons are equal and their relative azimuthal angle $\phi = 180^0$ is forbidden. Conversely in the $\pi^-\pi^+$ production the pions prefer to have a relative azimuthal angle 180^0 and the production of the pions propagating together is suppressed. The dominant one-body mechanism in both channels is related to the excitation of the Δ isobar. Hence the reaction can serve as a source of information about Δ's properties in nucleus. We have found that the reaction is sensitive to effects of the pion and Δ renormalization in the nuclear medium, similar to those found in the coherent (γ, π^0) reaction, but magnified because of the presence of the two pions.

1 INTRODUCTION

Coherent reactions of particle production in nuclei are interesting processes in many senses. They involve spin sums in the amplitudes and hence act as filters of the interaction selecting only some of the ingredients in the elementary transition matrix element. For instance, coherent pion production in (p, p') reactions in nuclei around the Δ region selects the spin-longitudinal part of the $NN \rightarrow N\Delta$ transition, while coherent photon production in the same region would select the spin-transverse part of the interaction. On the other hand, in the case of hadron production the results are sensitive to the final state interaction of the hadron with the nucleus, which allows one to obtain information on the hadron-nucleus optical potential. This is particularly useful if the produced hadron is a neutral one which cannot be used as a beam, like a π^0.

One of the traditional coherent production processes is the (γ, π^0) reaction, which has been the subject of intense study [1, 2, 3, 4, 5, 6, 7]. Here the results are rather sensitive to the way the pions and the Δ are renormalized in the nuclear medium and the reaction has been used as a test of the many-body theories which lead to the medium renormalization of such particles [8].

Coherent two pion photoproduction in nuclei has not been studied either theoretically nor experimentally, but the advent of new facilities like the Mainz Microtron and TJNAF, where $(\gamma, \pi\pi)$ reactions on the proton are been investigated [9, 10], opens reasonable expectations that such experiments could be undertaken soon. Some experimental attempts, so far inconclusive, have already been done [11].

The reactions we are discussing are

$$\gamma + A(g.s.) \rightarrow A(g.s.) + \pi^- + \pi^+ , \qquad (1a)$$

$$\gamma + A(g.s.) \rightarrow A(g.s.) + \pi^0 + \pi^0 , \qquad (1b)$$

where $A(g.s.)$ is the nucleus in the ground state. In the present work we calculate for the first time cross sections for the coherent two pion photoproduction. The fact that a realistic model has been developed which leads to a good reproduction of the experimental cross sections for the $(\gamma, \pi^-\pi^+)$ and $(\gamma, \pi^0\pi^0)$ reactions on the proton [12, 13], allows us to make predictions for the coherent $(\gamma, \pi\pi)$ reaction in those channels, which should be of use in the forthcoming experiments.

In addition to the reasons given above for the interest in the coherent (γ, π^0) reaction, the coherent $(\gamma, \pi\pi)$ reaction introduces new elements. It provides us with a unique opportunity for the study of the two pion photoproduction on spin-isospin zero objects. Here two pions are produced satisfying Bose-statistics and there are striking features which are tied to the isospin symmetry of the particles which would be interesting to observe. Eventual violations of this symmetry would also be interesting to the light of the much work devoted to this subject [14, 15, 16, 17].

On the other hand, the reaction acts as a filter of part of the interaction, thus offering new information on the elementary $(\gamma, \pi\pi)$ amplitudes, in the absence of a complete set of polarization observables, additional to the one obtained in present unpolarized experiments.

In the present work we shall develop the formalism and make predictions for coherent $(\gamma, \pi^-\pi^+)$ and $(\gamma, \pi^0\pi^0)$ reactions in ^{12}C.

2 Formalism

2.1 Symmetry properties and observables

The most general amplitude of the coherent two pion photoproduction on the spin-zero nucleus in the γA center of mass frame and working in the Coulomb gauge ($\vec{k} \cdot \vec{\epsilon} = 0$, $\epsilon^0 = 0$) is given by

$$F^{(\lambda)}(\vec{q}_1, \vec{q}_2; \vec{k}) = F_1(\vec{q}_1, \vec{q}_2; \vec{k})\, \hat{q}_1 \cdot \vec{\epsilon}_\lambda + F_2(\vec{q}_1, \vec{q}_2; \vec{k})\, \hat{q}_2 \cdot \vec{\epsilon}_\lambda , \qquad (2)$$

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where $\hat{q}_{1(2)} = \vec{q}_{1(2)}/q_{1(2)}$ is the unit vector for pion momentum, $\vec{k}$ is the photon momentum and $\vec{\epsilon}_\lambda$ with $\lambda = \pm 1$ is the photon polarization vector. In our study we will use a right-handed coordinate system in which the positive z-axis is along the photon momentum $\vec{k}$.

The main symmetry properties of the F_1 and F_2 amplitudes are connected with the isospin (isoscalar+isovector) structure of the electromagnetic current. In the case of the isospin-zero target, an isoscalar photon γ_S can produce two pions only in the state with isospin $T = 0$ (see Fig. 1a), which is necessarily symmetric. Bose-statistic requires that the orbital part of such system has to be symmetric too. Isovector photons γ_V produce pions only in the state with isospin $T = 1$ which requires an antisymmetrical orbital part. Thus, the amplitude for the coherent two pion photoproduction on isospin-zero targets can be divided into a symmetrical part $F_S^{(\lambda)}$, associated with scalar photons, and an antisymmetrical part $F_V^{(\lambda)}$, associated with vector photons:

$$F^{(\lambda)}(\vec{q}_1, \vec{q}_2; \vec{k}) = F_S^{(\lambda)}(\vec{q}_1, \vec{q}_2; \vec{k}) + F_V^{(\lambda)}(\vec{q}_1, \vec{q}_2; \vec{k}) \,. \tag{3}$$

The symmetry properties of the $F_S^{(\lambda)}$ and $F_V^{(\lambda)}$ amplitudes relative to the permutation $1 \leftrightarrow 2$ are the following

$$F_S^{(\lambda)}(\vec{q}_1, \vec{q}_2; \vec{k}) = F_S^{(\lambda)}(\vec{q}_2, \vec{q}_1; \vec{k}) \,, \qquad F_V^{(\lambda)}(\vec{q}_1, \vec{q}_2; \vec{k}) = -F_V^{(\lambda)}(\vec{q}_2, \vec{q}_1; \vec{k}) \,. \tag{4}$$

From Eq. (4) we can get the relations for the corresponding $F_1^{S,V}$ and $F_2^{S,V}$ amplitudes

$$F_1^S(\vec{q}_1, \vec{q}_2; \vec{k}) = F_2^S(\vec{q}_2, \vec{q}_1; \vec{k}) \,, \qquad F_1^V(\vec{q}_1, \vec{q}_2; \vec{k}) = -F_2^V(\vec{q}_2, \vec{q}_1; \vec{k}) \,. \tag{5}$$

In the case of the coherent $(\gamma, \pi^0\pi^0)$ reaction the pions can be produced only in the symmetric $T = 0$ state, but the $(\gamma, \pi^-\pi^+)$ channel contains contributions from both $T = 0$ and $T = 1$ pion isospin states. As we shall see below, this circumstance, combined with relations (5), will lead to an interesting physical phenomena.

Another peculiarity of the coherent two pion photoproduction is coming from total angular momentum and parity conservation. The total angular momentum of the initial photon-nuclear system for a spin-zero nucleus is necessarily $J > 0$. Since parity and total momentum are conserved, the pions orbital momenta $\vec{l}_1$ and $\vec{l}_2$ in the final state have to satisfy the following selection rules

$$\vec{l}_1 + \vec{l}_2 = \vec{J} > 0; \qquad l_1 + l_2 + J = even \,. \tag{6}$$

Thus, the minimal total momentum in the coherent double pion photoproduction on the spin-zero nuclei is $J_{min} = 1$. Therefore, the pions can not be produced both in the S state.

Additional symmetry properties are coming from the expressions for the differential cross section. As independent variables for the final three-particle state we shall take the pions solid angles Ω_1 and Ω_2 and the kinetic energy of one of the pions, for example, $T_1 = \omega_1 - m_\pi$. Then for the differential cross section we have

$$\frac{d\sigma}{dT_1 d\Omega_1 d\Omega_2} = \frac{S_B}{(2\pi)^5} \frac{q_1 q_2 M_A^2}{8 E_\gamma\, E_A\, E'_A} \frac{1}{|\, df/d\omega_2\,|} \frac{1}{2} \sum_\lambda |\, F_\lambda\,|^2 \,, \tag{7}$$

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where E_A (E'_A) is the nuclear energy in the initial (final) state, factor $S_B=1$ for $(\gamma, \pi^- \pi^+)$ and $S_B=1/2$ for $(\gamma, \pi^0 \pi^0)$ channels. The recoil factor $\mid df/d\omega_2 \mid$ in the phase space is given by

$$\mid df/d\omega_2 \mid = \mid 1 + \frac{\omega_2}{E'_A}(1 + \frac{\vec{q}_1 \cdot \vec{q}_2}{\vec{q}_2^{\,2}}) \mid \, . \tag{8}$$

Since in the case of the ^{12}C target $\frac{\omega_2}{E'_A} << 1$ we neglect the nuclear recoil effects and take

$$\mid df/d\omega_2 \mid \approx 1, \qquad E_A \approx E'_A \approx M_A, \qquad \omega_2 \approx E_\gamma - \omega_1 \, . \tag{9}$$

Then after averaging over photon polarization $\lambda = \pm 1$ we can express the differential cross section in terms of the F_1 and F_2 amplitudes from Eq. (2) as

$$\frac{d\sigma}{dT_1 d\Omega_1 d\Omega_2} = \frac{S_B}{(2\pi)^5} \frac{q_1 q_2}{16 E_\gamma} \{\mid F_1 \mid^2 \sin^2 \theta_1 + \mid F_2 \mid^2 \sin^2 \theta_2 +$$
$$2\, Re(F_1 F_2^*) \sin \theta_1 \sin \theta_2 \, \cos(\phi_1 - \phi_2) \} \, . \tag{10}$$

Since $F_{1(2)}$ are scalar functions they can depend upon $\vec{k} \cdot \vec{q}_1$, $\vec{k} \cdot \vec{q}_2$ and $\vec{q}_1 \cdot \vec{q}_2$, apart from the module of momenta. Hence $F_{1(2)}$ depends only on $\phi = \phi_1 - \phi_2$ and so does the unpolarized cross section of Eq. (7). Note that, the photon asymmetry Σ (for linear polarized photons) depends on ϕ_1 and ϕ_2 separately:

$$\Sigma\, d\sigma = -\frac{S_B}{(2\pi)^5} \frac{q_1 q_2}{16 E_\gamma} \{\mid F_1 \mid^2 \sin^2 \theta_1 \cos 2\phi_1 + \mid F_2 \mid^2 \sin^2 \theta_2 \cos 2\phi_2 +$$
$$2\, Re(F_1 F_2^*) \, \sin \theta_1 \sin \theta_2 \cos(\phi_1 + \phi_2) \} \, . \tag{11}$$

2.2 PWIA approach. Elementary amplitude

The expressions considered above are general and they do not include information about the reaction mechanisms. To calculate the nuclear amplitudes F_1 and F_2 we will follow the traditional way by applying as a first step the Plane Wave Impulse Approximation (PWIA). In such framework we assume that: i) in the final state the effects from the interaction between particles can be neglected and ii) the nuclear amplitude is a coherent sum of the elementary amplitude $t^{(\lambda)}$ (which describe the process on the free nucleon) averaged over nucleon distribution in nuclei

$$F_{PWIA}^{(\lambda)}(\vec{q}_1, \vec{q}_2; \vec{k}) \equiv V^{(\lambda)}(\vec{q}_1, \vec{q}_2; \vec{k}) = < \vec{q}_1, \vec{q}_2; 0 \mid \sum_{j=1}^{A} t^{(\lambda)}(\vec{q}_1, \vec{q}_2; \vec{k}, \vec{p}_j) \mid 0, \vec{k} >, \tag{12}$$

where $\mid 0 >$ is the nuclear ground state, the $\mid \vec{q}_1 >$, $\mid \vec{q}_2 >$ and $\mid \vec{k} >$ are pions and photon plane waves. Note that in the general case the elementary amplitude depends also upon the nucleon momentum $\vec{p}_j$ of the initial state. We shall take this dependence

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into account approximately applying the well-known folding procedure (factorization approximation) [7, 20]

$$\vec{p}_j \to \vec{p}_{eff} = -\frac{1}{2}(\vec{k} - \vec{q}_1 - \vec{q}_2)\,. \tag{13}$$

In the coherent two pion photoproduction on the spin-zero nuclei only the non-spin-flip part of the elementary amplitude contributes. Using the factorization approximation (13) it can be reduced to an expression with the same structure as Eq. (2), i.e.

$$t^{(\lambda)}(\vec{q}_1, \vec{q}_2; \vec{k}) = t_1(\vec{q}_1, \vec{q}_2; \vec{k})\,\hat{q}_1 \cdot \vec{\epsilon}_\lambda + t_2(\vec{q}_1, \vec{q}_2; \vec{k})\,\hat{q}_2 \cdot \vec{\epsilon}_\lambda\,. \tag{14}$$

Since we are dealing with isospin-zero nuclear targets (with equal numbers of protons and neutrons) we need only the isoscalar component of the elementary amplitude $t_{1(2)}$ (in the nucleon isospin space), which can be determined as

$$t_{1(2)} = \frac{1}{2}\left[t^p_{1(2)} + t^n_{1(2)}\right]\,, \tag{15}$$

where $t^p_{1(2)}$ and $t^n_{1(2)}$ are the amplitudes for the proton and neutron channels, respectively. We obtain the final expression for the $F_{1(2)}$ amplitudes in the PWIA approach by substituting Eq. (14) in Eq. (12):

$$F^{PWIA}_{1(2)}(\vec{q}_1, \vec{q}_2; \vec{k}) \equiv V_{1(2)}(\vec{q}_1, \vec{q}_2; \vec{k}) = A\,F_A(Q)\,t_{1(2)}(\vec{q}_1, \vec{q}_2; \vec{k})\,, \tag{16}$$

where A is the number of nucleons and $\vec{Q} = \vec{k} - \vec{q}_1 - \vec{q}_2$ is the transferred momentum, which is given by

$$Q^2 = k^2 + q_1^2 + q_2^2 - 2k(q_1 \cos\theta_1 + q_2 \cos\theta_2) + 2q_1 q_2 \cos\theta_{12}\,, \tag{17a}$$

$$\cos\theta_{12} = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \cos(\phi_1 - \phi_2)\,. \tag{17b}$$

The nuclear form factor $F_A(Q)$ (which is normalized to 1 at $Q=0$) is extracted from the well-known nuclear charge form factor using the relation $F^{ch.}_A(Q) = F_A(Q)f^{ch.}_p(Q)$ where $f^{ch.}_p$ is the proton charge form factor.

One of the basic elements in the expression of Eq. (16) is the elementary amplitude $t_{1(2)}$ which is the spin-isospin independent part of the total amplitude describing the process on the free nucleons. In the present paper we shall evaluate it using the recently developed model of Refs. [12, 13].

In the Ref. [12] the model for the $\gamma p \to \pi^- \pi^+ p$ reaction was described. It was constructed from effective Lagrangians involving the coupling of photons and pions to nucleons and resonances. The $\Delta(1232)$, $N^*(1440)$ and $N^*(1520)$ resonances were considered, as well as the coupling of ρ-mesons to the two pions system and 67 Feynmann diagrams were evaluated. In Ref. [13] the model was improved introducing the d-wave part of the $N^*(1520) \to \Delta\pi$ decay and other refinements. At the same time the number of diagrams was reduced to 20, which are shown in Fig. 2, once the relevant terms for energies below $E_\gamma = 800$ MeV were found in Ref. [12]. Also the model was extended

to the different isospin channels. We take thus the model of Fig. 2. Details on the Effective Lagrangians needed for their evaluation can be seen in Ref. [13] and further details on the explicit amplitudes in Ref. [18].

In the $\gamma p \to \pi^- \pi^+ p$ amplitude the dominant terms are the $\Delta N \pi \gamma$ Kroll-Ruderman term (i) which appears after the minimal substitution in the $\Delta N \pi$ vertex, the pion pole term (j) and the $N^*(1520)\Delta \pi$ term (p) which interferes with the $\Delta N \pi \gamma$ Kroll-Ruderman one leading to a peak in the cross section around $E_\gamma = 800$ MeV. For the case of the $\gamma p \to \pi^0 \pi^0 p$ reaction the $\Delta N \pi \gamma$ Kroll-Ruderman and pion-pole terms vanish and in this case other Δ terms and the $N^*(1520)\Delta \pi$ term give the largest contribution.

The $\Delta N \pi \gamma$ Kroll-Ruderman term is of an isovector nature, it contributes to $F_V^{(\lambda)}$. The averaged amplitude over proton and neutron of the diagram (i), considering the permutation of the π^+ and π^- in the two pion lines of the diagram, is given by

$$t^{(\lambda)} = e \left(\frac{2}{3} \frac{f^*}{m_\pi} \right)^2 \left[\frac{\vec{q}_{\pi^-}^* \cdot \vec{\epsilon}_\lambda}{\sqrt{s_{\Delta_1}} - M_\Delta + \frac{i}{2}\Gamma} - \frac{\vec{q}_{\pi^+}^* \cdot \vec{\epsilon}_\lambda}{\sqrt{s_{\Delta_2}} - M_\Delta + \frac{i}{2}\Gamma} \right] , \qquad (18)$$

where e is the proton charge, f^* is the $\pi N \Delta$ coupling and $\vec{q}_\pi^*$ is the pion momentum in the Δ rest frame. In the nuclear calculations performed in the γA c.m. frame we would express these momenta in terms of those in the γA c.m. frame by means of a boost [7]. Furthermore

$$s_{\Delta_1} = (p_f + p_{\pi^-})^2, \qquad s_{\Delta_2} = (p_f + p_{\pi^+})^2 . \qquad (19)$$

We can see that Eq. (18) has the structure of the general amplitude of Eq. (2) and the amplitude is of the $F_V^{(\lambda)}$ type, fulfilling the second of the Eq. (4). This is a consequence of the $T_3^\dagger$ operator in the $\gamma \Delta N \pi$ vertex, where $T_3^\dagger$ is the isospin 1/2 to 3/2 transition operator [13].

A trivial consequence is that, if $\vec{q}_{\pi^-}^* = \vec{q}_{\pi^+}^*$, the amplitude vanishes, which means that the dominant term in the elementary reaction gives null contribution when the π^-, π^+ are produced coherently and propagate together. In the case $\phi = \phi_1 - \phi_2 = 180^0$ (see Eq. (10)) the $\Delta N \pi \gamma$ Kroll-Ruderman term (i) and other terms of isovector nature will contribute maximally. Thus the strength of the cross section as a function of ϕ can be traced to a varying weight of the different terms of the elementary $(\gamma, \pi\pi)$ amplitude, wich gives chances to learn about the dynamics of the elementary process from the experimental measurements of such cross sections.

2.3   Final State Interaction (DWIA)

From the study of the nuclear single pion photoproduction (see for example Refs. [2, 3, 4, 5, 7]) it is well known that the interaction of the pions with the residual nucleus (Final State Interaction - FSI) is very important. At low energies the FSI increases the cross section and in the Δ-resonance region reduces it (approximately in about a factor 2). In the coherent two pion photoproduction we expect that FSI effects can be more important. The standard way to take into account FSI effects is to use the Distorted Wave Impulse Approximation (DWIA). In the present paper we shall develop this method in the momentum space to allow for a more accurate treatment of

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the nonlocal nature of the elementary operator. In such approach the DWIA amplitude can be presented in the form

$$F^{(\lambda)}(\vec{q}_1, \vec{q}_2; \vec{k}) = \int \Psi_{\vec{q}_1}(\vec{q}_1') \, \Psi_{\vec{q}_2}(\vec{q}_2') \, V^{(\lambda)}(\vec{q}_1', \vec{q}_2'; \vec{k}) \, d\vec{q}_1' \, d\vec{q}_2' \,. \tag{20}$$

Note that here we assumed that there is no pion-pion interaction. In the standard approach the pion distorted wave function $\Psi_{\vec{q}}(\vec{q}')$, which includes the effects of the pion-nuclear interaction, is a solution of the Klein-Gordon equation with a phenomenological optical potential. It can be expressed formally in terms of the pion-nuclear scattering amplitude $F_{\pi A}$:

$$\Psi_{\vec{q}}(\vec{q}') = \delta(\vec{q} - \vec{q}') - \frac{1}{(2\pi)^2} \frac{F_{\pi A}(\vec{q}, \vec{q}')}{\mathcal{M}(q')[E(q) - E(q') + i\epsilon]} \,, \tag{21}$$

where $E(q) = \omega(q) + E_A(q)$ is the total energy of the pion-nuclear system and $\mathcal{M}(q) = \omega(q)E_A(q)/E(q)$ is the pion-nuclear relativistic reduced mass.

In the present work the contributions from the FSI will be expressed in terms of the elastic pion-nuclear scattering amplitude, which was obtained as a solution of Lippmann-Schwinger equation with the phenomenological potential from Ref. [21]. After the substitution of Eq. (21) in Eq. (20) the final DWIA amplitude can be presented as a sum of the four terms

$$F^{(\lambda)}(\vec{q}_1, \vec{q}_2; \vec{k}) = V^{(\lambda)}(\vec{q}_1, \vec{q}_2; \vec{k}) + D_1^{(\lambda)}(\vec{q}_1, \vec{q}_2; \vec{k}) + D_2^{(\lambda)}(\vec{q}_1, \vec{q}_2; \vec{k}) + D_{12}^{(\lambda)}(\vec{q}_1, \vec{q}_2; \vec{k}) \,, \tag{22}$$

where

$$D_1^{(\lambda)}(\vec{q}_1, \vec{q}_2; \vec{k}) = -\frac{1}{(2\pi)^2} \int \frac{d\vec{q}_1'}{\mathcal{M}(q_1')} \frac{F_{\pi A}(\vec{q}_1, \vec{q}_1') \, V^{(\lambda)}(\vec{q}_1', \vec{q}_2; \vec{k})}{E(q_1) - E(q_1') + i\epsilon} \,, \tag{23a}$$

$$D_2^{(\lambda)}(\vec{q}_1, \vec{q}_2; \vec{k}) = -\frac{1}{(2\pi)^2} \int \frac{d\vec{q}_2'}{\mathcal{M}(q_2')} \frac{F_{\pi A}(\vec{q}_2, \vec{q}_2') \, V^{(\lambda)}(\vec{q}_1, \vec{q}_2'; \vec{k})}{E(q_2) - E(q_2') + i\epsilon} \,, \tag{23b}$$

$$D_{12}^{(\lambda)}(\vec{q}_1, \vec{q}_2; \vec{k}) = \frac{1}{(2\pi)^4} \int \frac{d\vec{q}_1' \, d\vec{q}_2'}{\mathcal{M}(q_1')\mathcal{M}(q_2')} \frac{F_{\pi A}(\vec{q}_1, \vec{q}_1')F_{\pi A}(\vec{q}_2, \vec{q}_2') \, V^{(\lambda)}(\vec{q}_1', \vec{q}_2'; \vec{k})}{[E(q_1) - E(q_1') + i\epsilon]\,[E(q_2) - E(q_2') + i\epsilon]} \,. \tag{23c}$$

The first term is the PWIA amplitude which was considered above. The second and third terms (D_1 and D_2) describe the processes when only one pion interacts with the residual nuclei. The last term D_{12} includes interaction of both pions.

In order to facilitate the numerical analysis it is convenient to introduce partial wave amplitudes. For the pion-nuclear scattering amplitude we will use standard partial wave amplitudes defined as

$$F_{\pi A}(\vec{q}, \vec{q}') = 4\pi \sum_{lm} \mathcal{F}_l(q, q') \, Y_{lm}(\Omega_{\hat{q}}) \, Y_{lm}^*(\Omega_{\hat{q}'}) \,, \tag{24}$$

where Y_{lm} are the spherical harmonics. For the coherent two pion photoproduction amplitudes we will use the following decomposition:

$$F^{(\lambda)}(\vec{q_1}, \vec{q_2}; \vec{k}) = \sum_{l_1 m_1} \sum_{l_2 m_2} \mathcal{F}^{(\lambda)}_{l_1 m_1 l_2 m_2}(q_1, q_2; k)\, Y_{l_1 m_1}(\Omega_1)\, Y_{l_2 m_2}(\Omega_2)\,, \qquad (25a)$$

$$V^{(\lambda)}(\vec{q_1}, \vec{q_2}; \vec{k}) = \sum_{l_1 m_1} \sum_{l_2 m_2} \mathcal{V}^{(\lambda)}_{l_1 m_1 l_2 m_2}(q_1, q_2; k)\, Y_{l_1 m_1}(\Omega_1)\, Y_{l_2 m_2}(\Omega_2)\,. \qquad (25b)$$

Some details for the numerical method developed for calculations of the partial amplitudes $\mathcal{V}^{(\lambda)}_{l_1 m_1 l_2 m_2}$ are given in the Appendix.

Finally let us consider one useful approximation in the treatment of the FSI. If in the pion nuclear Green's function

$$\frac{1}{E(q) - E(q') + i\epsilon} = \frac{P}{E(q) - E(q')} - i\pi\delta(E(q) - E(q')) \qquad (26)$$

we neglect the contribution from the principal value integral (first term in Eq. (26)) and take into account that

$$\mathcal{F}_l(q, q) = \frac{1}{2iq}(e^{2i\delta_l} - 1) \qquad (27)$$

then the FSI effects can be taken into account in a very simple way in terms of the pion-nuclear elastic scattering phase shifts δ_l, i.e.

$$\mathcal{F}^{(\lambda)}_{l_1 m_1 l_2 m_2}(q_1, q_2; k) = \mathcal{V}^{(\lambda)}_{l_1 m_1 l_2 m_2}(q_1, q_2; k)\, e^{i(\delta_{l_1} + \delta_{l_2})}\, \cos\delta_{l_1}\, \cos\delta_{l_2}\,. \qquad (28)$$

A similar expression can be obtained using the K-matrix approach. Therefore, we will refer to this approximation as "K-matrix" approximation. Below we will see that at high energies ($E_\gamma > 600$ MeV) this approximation is good not only for a qualitative but also for a quantitative description of the FSI effects.

3 Amplitude renormalization

In the previous considerations we used the conventional approach based on the Impulse Approximation neglecting the influence of the nuclear medium on the elementary process. However, in the study of pion nuclear elastic scattering and in the coherent (γ, π^0) reaction[1, 2, 3] it was found that the properties of the Δ resonance in the nuclear medium differ from those at zero matter density due to many-body corrections. In the present paper we shall take into account the corresponding Δ renormalization effects using the following modification of the Δ propagator

$$\left[\sqrt{s}_\Delta - M_\Delta + \frac{i}{2}\Gamma\right]^{-1} \rightarrow \left[\sqrt{s}_\Delta - M_\Delta + \frac{i}{2}\tilde{\Gamma}(\rho(r)) - \Sigma_\Delta(\rho(r))\right]^{-1}, \qquad (29)$$

where $\rho(r)$ is the nuclear density normalized to A, $\tilde{\Gamma}$ is the Pauli blocked width and Σ_Δ is the Δ selfenergy. The imaginary part of the Δ selfenergy includes contribution from the two- and three-body mechanisms of the $\Delta - h$ interaction and the real part takes into account the Landau-Migdal force $g'_\Delta (f^*/m_\pi)^2 \vec{S}\cdot\vec{S}^\dagger\,\vec{T}\cdot\vec{T}^\dagger$, which leads to the following renormalization effects [7]

$$Re\Sigma_\Delta \to Re\Sigma'_\Delta + \frac{4}{9}(\frac{f^*}{m_\pi})^2 \, g'_\Delta \, \rho(r) \tag{30}$$

with g'_Δ=0.55. Parametrizations for $\tilde{\Gamma}_\Delta$, $Re\Sigma'_\Delta$ and $Im\Sigma_\Delta$ can be found in Refs. [7, 22]. Our analysis and calculations in Ref. [7] show that such renormalization of the Δ propagator provides a good description of the coherent (γ, π^0) reaction on ^{12}C. This circumstance gives us confidence in the study of the medium effects in the coherent two pion photoproduction.

4   Results and Discussion

Symmetry properties. In the case of the two pion photoproduction, in contrast to the single pion photoproduction, we have more independent kinematical variables in the final state and, therefore, more possibilities for the study of the mechanisms of this reaction. One of the interesting kinematical regions corresponds to the case when two pions propagate together, i.e. in the same direction and with equal energies. In Fig. 3 we present results of the PWIA analysis at $\theta_1 = \theta_2 = 15^0$ and at $\phi = \phi_1 - \phi_2 = 0, 90^0$ and 180^0. In the case of $\phi = 0$ the pions propagate in the same direction. In the case of $\phi = 180^0$ they are in the same plane with the initial photon and in opposite sides of it.

One of the main conclusions which follows from the results depicted in Fig. 3a is that the probability of propagation of π^+- and π^--mesons together (which corresponds to the middle of the energy distribution at $\phi = 0$) is very small (about 0.1 nb). They prefer to propagate in one plane with the photon with $\phi = 180^0$. In the $(\gamma, \pi^0\pi^0)$ channel (see Fig. 3b) the situation is reversed. The two π^0 mesons prefer to propagate together. However, the cross section in this kinematical region is about three orders of magnitude smaller than in the $(\gamma, \pi^-\pi^+)$ channel when the charged pions go out with $\phi = 180^0$. In order to understand such behaviour let us recall the symmetry properties considered in Section 1. From Eq. (10) we have

$$\frac{d\sigma}{dT_1 d\Omega_1 d\Omega_2}(\phi = 0) = \frac{S_B}{(2\pi)^5} \frac{q_1 q_2}{16E_\gamma} \mid F_1 \sin\theta_1 + F_2 \sin\theta_2 \mid^2, \tag{31a}$$

$$\frac{d\sigma}{dT_1 d\Omega_1 d\Omega_2}(\phi = 180^0) = \frac{S_B}{(2\pi)^5} \frac{q_1 q_2}{16E_\gamma} \mid F_1 \sin\theta_1 - F_2 \sin\theta_2 \mid^2. \tag{31b}$$

In the coherent $(\gamma, \pi^0\pi^0)$ reaction on the isospin-zero nuclei only isoscalar photons give the contribution in the cross section and they create two pions in the symmetric state with isospin $T = 0$. Therefore, as it follows from Eq. (5), $F_1(\vec{q}_1, \vec{q}_2; \vec{k}) = F_2(\vec{q}_2, \vec{q}_1; \vec{k})$ and the maximum cross section is in the region where $\vec{q}_1 = \vec{q}_2$ (two pions propagate together).

In the $(\gamma, \pi^-\pi^+)$ channel both isoscalar and isovector photons contribute in the cross section, but from the analysis of the $\pi^0\pi^0$ photoproduction it follows that the role of the isoscalar photons is very small. Therefore, the dominant contribution in the $\pi^-\pi^+$ channel comes from the isovector photons which produce pions in the antisymmetric

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state with isospin $T = 1$. Since in this case (see Eq. (4)) $F_1(\vec{q}_1, \vec{q}_2; \vec{k}) = -F_2(\vec{q}_2, \vec{q}_1; \vec{k})$ the differential cross section has a dip minimum at $\vec{q}_1 = \vec{q}_2$ region which is filled only by small contributions from the isoscalar photons. For the same reason the maximum cross section is found when $\phi = 180^0$.

Note that this interesting feature of the coherent two pion photoproduction is based only on the isospin symmetry properties and it does not depend on the mechanisms of the process or FSI effects. If an experimental study would find serious deviations from our predictions (for example, nonzero cross section in the kinematical region where energies and polar angles of the two π^0 mesons are equal and their relative azimuthal angle $\phi = 180^0$) this would be an indication of isospin violation effects.

The study of the elementary amplitude. Another feature is related directly to the dynamical aspects of the reaction. In Fig. 4 the energy distributions of the π^- mesons calculated in the PWIA approach at E_γ=450, 600 and 750 MeV are depicted. Here we analyse the contributions coming from different parts of the elementary amplitude (see also Fig. 2): Born terms (diagrams (a)-(h)), Δ isobar terms (diagrams (i)-(o)) and contributions from $D_{13}(1520)$ and $P_{11}(1440)$ baryon resonances (diagrams (p)-(t)). In the $(\gamma, \pi^-\pi^+)$ channel the contribution of the Born terms is very small (about 1-2%) in a large photon energy region. The dominant contribution is related to the excitation of the Δ isobar (dashed curves in Fig. 4). The contributions coming from the $P_{11}(1440)$ and $D_{13}(1520)$ resonances tend to cancel each other. In the $(\gamma, \pi^0\pi^0)$ channel the situation is similar with only one exception: the Born terms at low energies $(E_\gamma < 450\text{MeV})$ are also important.

Finally, note that the energy distribution in the $(\gamma, \pi^0\pi^0)$ channel is exactly symmetrical relative to point where the energies of the pions are equal $(T_{\pi_1} = T_{\pi_2})$. However in the $(\gamma, \pi^-\pi^+)$ channel this symmetry is slightly destroyed due to the interference of the small contribution from the isoscalar photons with the dominant contributions from isovector photons.

Final State Interaction effects. The treatment of the FSI effects in the case of the two pion photoproduction is more complicated in contrast to the single pion photoproduction case. From the study of the last one [2, 3, 4, 5, 7] it is well known that FSI increases the differential cross section at low energies $(T_\pi < 100)$ MeV and reduces it in the Δ resonance region (in about a factor two).

However, in the two pion photoproduction if one pion has low energy the second pion can be produced in the Δ resonance region. Therefore, the FSI of one pion can be canceled by the FSI of the second one. This peculiarity of the FSI is illustrated in Fig. 5 for the case of $E_\gamma = 450$ MeV. If only one pion is interacting with the residual nucleus, then the FSI leads to the enhancement of the cross section in the low-energy part of the phase space and to the reduction in the high-energy part (compare the dotted curve with the dashed or with the dash-dotted curves). In the full calculations (solid curve) the FSI increase the cross section only in the region where both pions are at relatively low energies (in the middle of the phase space). Note that due to the Coulomb interaction the FSI effects are larger for low-energy π^- mesons than for the low-energy π^+ mesons. This produces an additional asymmetry in the energy distribution relative the point where the pions have equal energy.

The off-shell effects. The FSI contributions contain one ingredient which is not

well defined. This is the propagation of the pions in the off-shell region where $\omega \neq \sqrt{m_\pi^2 + \vec{q}^{\,2}}$. In terms of the pion-nuclear Green's function (26) it corresponds to contributions coming from the principal value integral. In this part we have to define the elementary amplitude $t^{(\lambda)}$ also for the photoproduction of off-shell pions.

On the other hand the pion-nuclear scattering amplitude $F_{\pi A}(\vec{q}\,', \vec{q})$ which is normally extracted from the pion-nuclear elastic scattering data is well defined only in the on-shell region (at $q' = q$). Its off-shell behavior, which specifies the dynamics of the pion-nuclear system in the interacting region (inside the nucleus), depends on the extrapolation of the elementary pion-nucleon amplitude to the off-shell region. The off-shell dependence of these amplitudes can have repercussion in production processes. In coordinate space this could be stated as saying that optical potentials which provide the same scattering properties can lead to different results in the production process.

To study the sensitivity of the two pion photoproduction to these ingredients of the theory we shall use the standard prescription for the extrapolation of the elementary amplitude in the off-shell region i.e.

$$t^{(\lambda)}(\vec{q_1}, \vec{q_2}; \vec{k}) \rightarrow t^{(\lambda)}(\vec{q_1}, \vec{q_2}; \vec{k}) \, F(q_1) \, F(q_2) \,, \tag{32}$$

where for the form factor $F(q)$ we use monopole type expression

$$F(q) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - \omega^2 + \vec{q}^{\,2}} \,. \tag{33}$$

Note that this procedure can be interpreted also as a modification of the pion-nuclear scattering amplitude $F_{\pi A}$ in the off-shell region without changing its on-shell values (in other words, to use a phase-equivalent optical potentials which provide different behavior of the pion wave function in the interaction region and the same asymptotic).

In Fig. 6, for the photon energy 450 MeV, we illustrate the sensitivity of the FSI contributions to the different values of the cut-off parameter Λ=500, 1000 and 1500 MeV. We see that the DWIA results caused by different off-shell extrapolations can differ by about 50%. However, with increasing photon energy the contribution from the off-shell region reduces (compare solid and dashed curves). Starting from E_γ=600 MeV the simple "K-matrix" approximation (see Eq. (28)), where we deal only with on-shell pions, becomes a reliable approach to account for FSI effects. In this region they are very important and can be described only with pion-nuclear scattering phase shifts.

Delta renormalization effects. One of the important problem which can be studied in the coherent two pion photoproduction is the modification of the elementary operator in the nuclear medium. From the analysis of the elementary amplitude, we have seen above that the main one-body mechanism is related to the excitation of the Δ-isobar. This finding allows us to do reliable investigations of the Δ-renormalization effects, as it was done in the coherent single π^0-photoproduction [2, 3, 7].

In Fig. 7 we compare the conventional DWIA calculations presented above with the results obtained including the renormalization of the Δ-propagators in accordance with Eqs. (29,30). Note that the corresponding medium effects are different in the different

kinematical regions. They are maximal in the region where one of the pions is in the Δ-resonance region (around $T_\pi = 180$ MeV) and become less relevant at lower and higher energies. Due to this fact, the renormalization of the Δ-propagators changes not only the absolute value of the differential cross section, but due to the presence of the two pions it also modifies the shape of the energy distribution. The most sizeable effects are seen at $E_\gamma = 750$ MeV. Thus we have found that medium effects in the coherent two pion photoproduction are very important and can be observed experimentally.

Angular correlation and total cross section. In the next Fig. 8 the dependence on the pion polar angle θ (angular correlation) is depicted both for the $\pi^-\pi^+$ and $\pi^0\pi^0$ channels. From the discussions of the symmetry properties it follows that the differential cross section reaches the maximal value in the middle of the phase space at $\phi = 0$ in the $\pi^0\pi^0$ and at $\phi = 180^0$ in the $\pi^-\pi^+$ channels (see also Fig. 3). We have done the calculations for these kinematical regions assuming that $\theta_1 = \theta_2$.

One of the peculiarities which follows from the general symmetry properties is that the angular correlation function vanishes at $\theta_1 = \theta_2 = 0$ or 180^0 due to the $\sin\theta_{1(2)}$ dependence of the differential cross section. Furthermore, in the $\pi^0\pi^0$ channel we are already in the region where the nuclear form factor has a minimum. This is because at $\phi = 0$ the transferred momentum is larger than at $\phi = 180^0$ (see Eq. (17)).

The energy dependence of the total cross section (integrated over the whole phase space) is depicted in Fig. 9. Here we compare the PWIA , DWIA and results obtained with Δ renormalization. At $350 < E_\gamma < 450$ MeV the FSI can increase the cross section up to a factor two. Note that such enhancement is smaller than what was found in the case of inclusive double photoproduction [19]. The reason must be traced to the fact that, in contrast to the coherent photoproduction, in the inclusive reaction, due to the energy transfer to the residual nucleus, both pions can be in the low-energy region where the enhancement from the πA interaction shows up. At $E_\gamma > 500$ MeV FSI reduces the total cross section up to a factor 4-5. The further reduction comes from the Δ renormalization effects.

Finally, let us make one comment about the $\pi^0\pi^0$ channel. We have found that in this channel the cross section is about 1000 times smaller than in the $\pi^-\pi^+$ channel. In such a situation we expect that in the $\pi^0\pi^0$ channel the contribution of the pion rescattering on the different nucleons with charge exchange could be important. Note that similar effects have been found in the single π^0 photoproduction on the deuteron [23, 24]. Moreover, recently in the $(\gamma, \pi^0\pi^0)$ reaction on the proton [25] at threshold region it was shown that such rescattering effects (on the same nucleon) are important. The role of the coupling to the charge-exchange channel mentioned above is beyond the DWIA approach and could be the subject of further investigations. We expect that due to this mechanism, the $\pi^0\pi^0$ channel could be appreciably enhanced.

On the other hand the fact that the two π^0 mesons come mostly together and in a $T = 0$ state could be used to investigate possible renormalization effects of such pionic state, which have been suggested in Refs. [26, 27].

5 Conclusions

Using a recently developed model for the two pion photoproduction on the nucleons based on Effective Lagrangians we have studied the coherent $(\gamma, \pi^- \pi^+)$ and $(\gamma, \pi^0 \pi^0)$ reactions in ^{12}C. The coherence of these reactions and its study in isospin-zero nuclei has allowed us to see interesting effects tied to the bosonic symmetry of the pions which do not depend on the reaction mechanisms.

We have found that in the $(\gamma, \pi^- \pi^+)$ channel the dominant contribution comes from the isovector part of the electromagnetic current. In this case the probability of production of π^-- and π^+-mesons propagating together is very small. On the other hand, only the isoscalar electromagnetic current gives the contribution in the $(\gamma, \pi^0 \pi^0)$ channel. In this case, due to isospin symmetry arguments, two π^0-mesons prefer to propagate together. On the other hand, the kinematical region where the energies and polar angles of the two π^0 mesons are equal and their relative azimuthal angle $\phi = 180^0$ becomes forbidden if the total isospin is a good quantum number.

The main one-body mechanism in both channels is related to the excitation of the Δ isobar. This finding could be used in future investigations of the renormalization of the Δ propagator in the nuclear medium, as it was done in the coherent (γ, π^0) reaction. Many-body effects caused by pion rescattering (or FSI) and Δ renormalization are very important. We have found that near threshold FSI can increase the cross section in about a factor two. At $E_\gamma > 600$ MeV it reduces the cross section in about a factor 4-5. In this region the rescattering of the on-shell pions is dominant and with good accuracy the FSI effects can be described using simple "K-matrix" approximation. The Δ renormalization leads to an important reduction of the cross section and it modifies the shape of the energy distribution of the pions.

Appendix

In this Appendix we present some details for the calculations of the partial amplitudes $\mathcal{V}^{(\lambda)}_{l_1 m_1 l_2 m_2}$. In accordance with Eq. (25) they are defined as

$$\mathcal{V}^{(\lambda)}_{l_1 m_1 l_2 m_2}(q_1, q_2; k) = \int V^{(\lambda)}(\vec{q}_1, \vec{q}_2; \vec{k}) \, Y^*_{l_1 m_1}(\Omega_1) \, Y^*_{l_2 m_2}(\Omega_2) \, d\Omega_1 \, d\Omega_2 \,. \qquad (A1)$$

In order to perform the integration over the pions azimuthal angles ϕ_1 and ϕ_2 we explore the fact that the $V_{1(2)}$ amplitudes depend on the relative azimuthal angle $\phi = \phi_1 - \phi_2$. Therefore, we can introduce a fastly convergent expansion

$$V_{1(2)}(\vec{q}_1, \vec{q}_2; \vec{k}) = \sum_m V^{(m)}_{1(2)}(q_1, q_2, x_1, x_2; k) \, e^{im\phi} \qquad (A2)$$

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where $x_{1(2)} = \cos\theta_{1(2)}$. The inverse expression is given by

$$V_{1(2)}^{(m)}(q_1, q_2, x_1, x_2; k) = \frac{1}{2\pi} \int_0^{2\pi} V_{1(2)}(\vec{q}_1, \vec{q}_2; \vec{k})\, e^{-im\phi}\, d\phi. \tag{A3}$$

Using Chebyshev quadrature for numerical integration in Eq. (A3) for the $V_{1(2)}^{(m)}$ amplitude we obtain

$$V_{1(2)}^{(m)}(q_1, q_2, x_1, x_2; k) = \frac{1}{N} \sum_{i=1}^{N} V_{1(2)}(\vec{q}_1^{\,i}, \vec{q}_2^{\,i}; \vec{k})\, Re[e^{-im\phi_i}], \tag{A4}$$

where $\cos\phi_i = \cos\left[\frac{(2i-1)\pi}{N}\right]$ and $\vec{q}_{1(2)}^{\,i}$ are the corresponding pion momenta.

After the integration over ϕ_1 and ϕ_2 angles in Eq. (A1) the partial amplitudes $\mathcal{V}_{l_1 m_1 l_2 m_2}^{(\lambda)}$ can be expressed in terms only of the $V_{1(2)}^{(m)}$ amplitudes:

$$
\begin{aligned}
\mathcal{V}_{l_1 m_1 l_2 m_2}^{(\lambda)} =\ & -\lambda\,(2\pi)^{3/2} \int_{-1}^{1} \int_{-1}^{1} dx_1\, dx_2\, \left[V_1^{(m_1-\lambda)} \sin\theta_1 + V_2^{(m_1)} \sin\theta_2 \right] \\
& \times Y_{l_1 m_1}(x_1, \phi_1 = 0)\, Y_{l_2 m_2}(x_2, \phi_2 = 0),
\end{aligned} \tag{A5}
$$

where $m_2 = -m_1 + \lambda$. For the numerical integration in Eq. (A5) we use the Gaussian quadrature.

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Figure captions

Fig. 1. Isospin selection rules for the coherent two pion photoproduction on the isospin zero targets.

Fig. 2. Feynman diagrams of the model of Ref.[13] for the reaction $\gamma N \to N\pi\pi$.

Fig. 3. Energy spectra of the π^- **(a)** and π^0 **(b)** mesons in the $(\gamma, \pi^-\pi^+)$ and $(\gamma, \pi^-\pi^+)$ channels, respectively, at $E_\gamma = 600$ MeV and $\theta_1 = \theta_2 = 15^0$. Dashed, dash-dotted and solid curves are the PWIA calculations at $\phi = 0, 90$ and 180^0, respectively.

Fig. 4. Energy spectra (integrated over pions solid angles) of the π^- **(a)** and π^0 **(b)** mesons in the $(\gamma, \pi^-\pi^+)$ and $(\gamma, \pi^0\pi^0)$ channels, respectively. at $E_\gamma = 450, 600$ and 750 MeV. Dashed, dotted and solid curves are the PWIA calculations with Δ, $\Delta + D_{13}$, and $\Delta + Born + P_{11} + D_{13}$ terms, respectively.

Fig. 5. FSI effects in the $(\gamma, \pi^-\pi^+)$ channel at $E_\gamma = 450$ MeV. The dotted curve is the PWIA result. The dashed and dash-dotted curves are the DWIA calculations when only π^-- or π^+-mesons are interacting with the residual nucleus, respectively. The solid curve is the full DWIA results obtained with Λ=1 GeV.

Fig. 6. Off-shell effects in the FSI. The dotted curves are the PWIA results. The dashed curves are the results obtained using the "K-matrix" approximation (see Eq. (28)). At $E_\gamma = 450$ MeV the upper, middle and lower solid curves are the full DWIA results with $\Lambda = 1.5, 1.0$ and 0.5 GeV, respectively. At E_γ=600 and 750 MeV the solid curves are full DWIA results with $\Lambda = 1.0$ GeV.
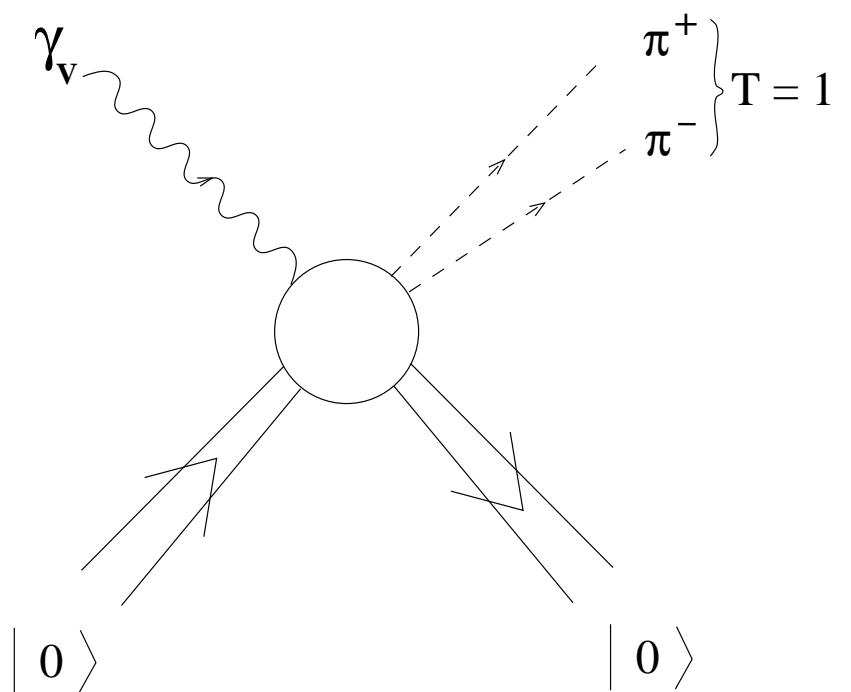
Fig. 7. Δ renormalization effects. The dashed curves are the DWIA (with $\Lambda = 1$ GeV) results. The solid curves are the results obtained with renormalized Δ propagator (see Eqs. (29,30)).

Fig. 8. Angular distribution of the π^--mesons in the $(\gamma, \pi^-\pi^+)$ reaction at $\phi = 180^0$ and of the π^0-mesons in the $(\gamma, \pi^0\pi^0)$ reaction at $\phi = 0$ in the middle of the phase space. The dotted and dashed curves are the PWIA and full DWIA (with $\Lambda = 1$ GeV) calculations, respectively. The solid curves are the results obtained with renormalized Δ propagator.

Fig. 9. Cross section for the coherent $(\gamma, \pi^-\pi^+)$ and $(\gamma, \pi^0\pi^0)$ reactions on ^{12}C as a function of photon energy. Notations of the curves are the same as in Fig. 8.
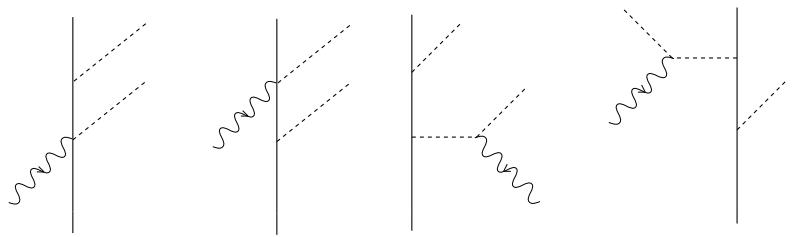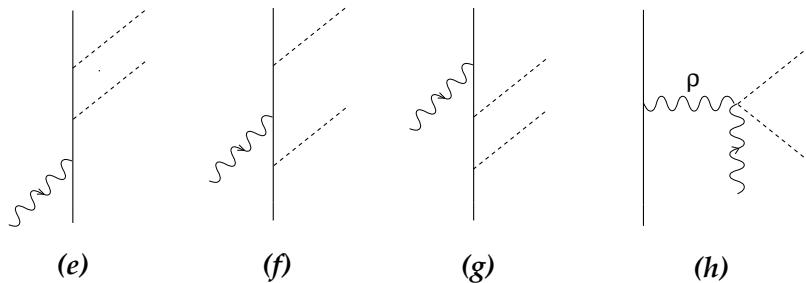
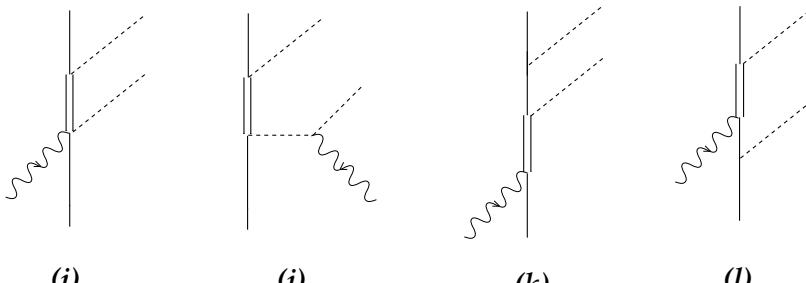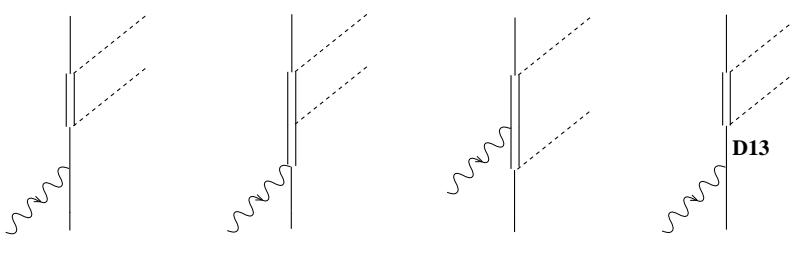γ_s

$\left.\begin{array}{c}\pi^{+,\,0}\\\\\pi^{-,\,0}\end{array}\right\}T=0$

$|0\rangle$ $|0\rangle$

(a)

γ_v

$\left.\begin{array}{c}\pi^{+}\\\\\pi^{-}\end{array}\right\}T=1$

$|0\rangle$ $|0\rangle$

(b)

Fig. 1

(a)   *(b)*   *(c)*   *(d)*

(e)   *(f)*   *(g)*   *(h)*

(i)   *(j)*   *(k)*   *(l)*

(m)   *(n)*   *(o)*   *(p)*

(q)   *(r)*   *(s)*   *(t)*

Fig. 3a

$^{12}C(\gamma, \pi^0\pi^0)^{12}C$

PWIA

$E_\gamma = 600$ MeV

$\theta_1 = \theta_2 = 15^0$

(b)

$T_{\pi 0}$(MeV)

$d\sigma/dT_\pi [nb/MeV/(sr)^2]$

Fig. 3b

Fig. 4a

$^{12}C(\gamma, \pi^0\pi^0)^{12}C$

PWIA

$E_\gamma = 450$ MeV

$E_\gamma = 600$ MeV

$E_\gamma = 750$ MeV

(b)

$d\sigma/dT_\pi$(nb/MeV)

$T_{\pi 0}$(MeV)

Fig. 4b

$^{12}C(\gamma, \pi^- \pi^+)^{12}C$

$E_\gamma = 450$ MeV

Fig. 5

$^{12}C(\gamma,\pi^-\pi^+)^{12}C$

DWIA

$E_\gamma = 450$ MeV

$E_\gamma = 600$ MeV

$E_\gamma = 750$ MeV

$d\sigma/dT_\pi \, (\mu b/MeV)$

T_{π^-} (MeV)

Fig. 6

Fig. 7

$(\gamma,\pi^-\pi^+)$, $\phi=180^0$ $T_1=T_2=167$ MeV

$E_\gamma=600$ MeV

$(\gamma,\pi^0\pi^0) \times 10$, $\phi=0$

$d\sigma/dT_\pi[\mu b/MeV/(sr)^2]$

$\theta_1=\theta_2$ (deg)

Fig. 8

Fig. 9